STEADY AND UNSTEADY GRADUALLY VARIED FLOW CALCULATION FOR GRAVITY PIPES USING MICROSOFT EXCEL

Nabil A. Zaghloul and Mohamed Abd El-Ghany
Department of Civil Engineering, Kuwait University, P.O. Box 5969, Safat 13060, Kuwait

ABSTRACT

The length of the steady Gradually Varied Flow (G.V.F) profile in a circular gravity pipe section is computed using the Graphical Integration Method. The equations used for the solution are: a) the dynamic equation of Gradually Varied Flow in a prismatic channels, b) the hydraulic exponents M and N equation derived by Chow [2], and c) the Varied hydraulic exponents M (y/d_o) and $N(y/d_o)$ equation modified by Zaghloul [15]. The results of the calculated G.V.F profile length using the modified hydraulic exponents $M(y/d_o)$ and $N(y/d_o)$ equation are closer to the G.V.F length calculated based on the exact formulation of the G.V.F dynamic equation. The percentage difference ranges from 0.67% to 8.72% for various bed slopes and G.V.F depth limits. The calculated G.V.F profile length using the Chow hydraulic exponents M and N resulted in wider values with percentage difference ranges from 0.16 to 25.59%. Hence, a remarkable improvement of the computation of G.V.F profile is achieved using the modified $M(y/d_o)$ and $N(y/d_o)$ hydraulic exponents.

The unsteady Gradually Varied Flow wave propagation in circular gravity pipe section is simulated using the Explicit Method. The Extended Transport block (EXTRAN) of the latest Storm Water Management Model (PCSWMM2000) was used to route the wave through a circular gravity pipe section. The resulting routed hydrographs by the Explicit Method and the EXTRAN Block of the PCSWMM 2000, provided similar flow peak and lag time in both cases.

A computer package was developed for the Steady G.V.F length calculation for gravity pipes using the Microsoft Excel spreadsheet. The results are plotted using the Excel graphics capabilities.

A second computer package using Microsoft Excel was developed for the Unsteady G.V.F simulation based on the Explicit Method. The routed hydrographs are plotted by the Excel graphics capabilities.

The Excel packages for the Steady and Unsteady G.V.F are user friendly and are posted on the Internet Website location http://briefcase.yahoo.com/civil_engineering2001. A Read me File is provided for each Excel package and is used as a users guide.

Keywords: steady gradually varied flow, graphical integration method, unsteady gradually varied flow, the Saint-Venant equations, hod, flow routing, Urban drainage and sewerage systems.

INTRODUCTION

Non-uniform flow in a prismatic channel with gradual changes in its free surface elevation is termed Gradually Varied Flow, G.V.F. The temporal variation of the flow could result in steady or unsteady G.V.F. Design of wastewater collection and urban drainage systems involves the computation of the steady and/or unsteady G.V.F profiles. Most of these systems consist largely of circular pipes.

The steady uniform flow regime is commonly assumed in designing the size of the pipe diameter based on the uniform flow formulae [3,5]. Presence of weirs, drop manholes, outfalls, converging and diverging sewer systems and changing bed slopes may generate a steady rapidly varied flow (R.V.F) and/or a steady gradually varied flow (G.V.F). Hydraulic jump is a typical example of a (R.V.F) condition. The momentum principal governs the calculation of the hydraulic jump. Backwater profiles are typical examples of the (G.V.F) condition and the application of the energy principal governs the calculation of the energy head loss. Almost all major hydraulic engineering activities in open channel flow involve the computation of the G.V.F profile length.

The computation of the G.V.F profile involves the determination of the hydraulic exponents, M and N for critical and uniform flows, respectively. Forms for these hydraulic exponents were proposed by Bakhmetaff [1], Matzke [10], Mononobe [11], Von Seggern [13], and Chow [2]. The classical direct integration method for the solution of the dynamic equation of the G.V.F was suggested by Chow [2,3]. The method assumed a constant value of the hydraulic exponents M and N based on the averaged G.V.F depth for the reach. Zaghloul [15] developed the varied hydraulic exponents $M(y/d_o)$ and $N(y/d_o)$ for circular channel section to provide accurate computation of the G.V.F length.

The computation of the G.V.F profile length is calculated using the hydraulic exponents M and N for varying and constant averaged G.V.F depth for various bed slopes and G.V.F depth limits. The Graphical Integration Method is used to compute the G.V.F length. The results of the calculated G.V.F profile length are compared to the G.V.F length calculated from the exact formulation of the G.V.F dynamic equation. The results are presented in a tabular and/or graphical form. A computer package using Microsoft Excel spreadsheet is developed to calculate the G.V.F length for fixed and minimum depth limits and various bed slopes. The G.V.F profiles are plotted using the Excel graphics capabilities.

The temporal variation of the G.V.F is termed unsteady G.V.F. Design of wastewater collection and urban drainage systems, which largely consist of circular pipes, generally involves computation of unsteady G.V.F profiles. The numerical solution of the unsteady G.V.F profiles is conducted using the Explicit Method [8, 9, 14] and the Extended Transport Block of the Storm Water Management Model (PCSWM2000) which is the latest Model Version [6, 7].

A second computer package using Microsoft Excel is developed to simulate the unsteady G.V.F profile of the flood wave propagation in a circular gravity pipes. The computer package utilizes the numerical solution of the Explicit Method. The simulated hydrographs are plotted using the Excel graphics capabilities.

The routed hydrographs using the Excel package are compared with simulated hydrographs using the Extended Transport Block (EXTRAN) of the latest version of the Storm Water Management Model (PCSWMM 2000). The resulting hydrographs from the Excel package and the EXTRAN block are close in terms of peak discharge and lag time.

The Excel packages for steady and unsteady G.V.F profile are user friendly and easy to use. The two packages are posted on the Internet Website Location http://briefcase.yahoo.com/civil_engineering2001. Two Read me Files are provided as a users guide for the packages.

THE HYDRAULIC EXPONENT FOR CRITICAL FLOW COMPUTATION (M)

Chow [2] considered the section factor for critical flow computation Z is related to the G.V.F depth y, as follows

$$Z^2 = Cy^M \tag{1}$$

where,
M is the hydraulic exponent for critical flow condition using constant averaged
    G.V.F depth, and
C is a constant coefficient that depends on the section geometry.

Considering $Z^2 = A^3/T$, eqn. (1) produces an expression for M, namely,

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right) \tag{2}$$

where,

y is the G.V.F depth,
A is the area of cross section,
T is the top width.

For a circular section, M can be expressed as

$$M = \frac{(y \cdot d_0)}{(A \cdot d_0^2)}\left[3\left(T/d_0\right) - \frac{(A \cdot d_0^2)}{(T \cdot d_0)^2}\left\{1 - 2\left(y/d_0\right)\right\}\right] \tag{3}$$

where d_0 is the pipe diameter.

It is apparent from equation (3), that M depends on the relative depth (y/d_0). Zaghloul [15], modified the expression of M to vary continuously with relative depth (y/d_0). Thus,

$$M\left(y/d_o\right) = \frac{1}{\ell n\left(y \cdot d_o\right)}\left[\ell n\left[\frac{(A \cdot d_o^2)^3}{(T \cdot d_o)}\right] + C_1\right] \tag{4}$$

where,

$$C_1 = M\left(y/d_o\right) - \ell n \left[\frac{\left(A\ d_o^2\right)^3}{\left(T\ d_o\right)} \right] \tag{5}$$

with the initial value of $M\left(y/d_o\right) = 4.0974$ at $\left(y/d_o\right) = 0.01$.

THE HYDRAULIC EXPONENT FOR UNIFORM FLOW COMPUTATION (N)

Chow [2] introduced the channel conveyance K as a function of the G.V.F depth y, namely

$$K^2 = C'\, y^N \tag{6}$$

where,

N is the hydraulic exponent for uniform flow condition using constant averaged
     G.V.F depth, and

C' is a constant coefficient depending on channel geometry and roughness.

Following the same procedure outlined in the development of the hydraulic exponent, M, it can be shown that

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{7}$$

where,

P is the wetted perimeter,

R is the hydraulic radius.

For a circular section, N can be expressed as

$$N = \frac{2\left(y\ d_o\right)}{3\left(A\ d_o^2\right)}\left[5\left(T/d_o\right) - \frac{4\left(R\ d_o\right)}{\left(T\ d_o\right)}\right] \tag{8}$$

Zaghloul [15], modified the expression of the hydraulic Exponent N to vary with the relative G.V.F depth (y/d_o). Thus,

$$N\left(y/d_o\right) = \frac{1}{\ell n\left(y\ d_o\right)}\left[\ell n\left[\left(\frac{A}{d_o^2}\right)^2\left(\frac{R}{d_o}\right)^{4/3}\right] + C_2\right] \tag{9}$$

where,

$$C_2 = N\left(y/d_o\right)\ell n\left(y/d_o\right) - \ell n\left[\left(\frac{A}{d_o^2}\right)^2\left(\frac{R}{d_o}\right)^{4/3}\right]$$ (10)

with the initial value of $N(y/d_o) = 4.422$ at $(y/d_o) = 0.01$.

COMPUTATION OF STEADY GRADUALLY VARIED FLOW

Although there are a variety of solution techniques for accomplishing the computation of the steady G.V.F profile, given a particular situation, one method may be superior to the others. All solutions of the steady G.V.F equation must begin with the depth of flow at a control and proceed in the direction in which the control operates.

Three methods are discussed here: namely the exact dynamic gradually varied equation, the hydraulic exponents M and N as introduced by Chow [2], and the modified hydraulic exponents $M(y/d_o)$, as derived by Zaghloul [15].

The length of the G.V.F profile is calculated for the above three methods using the Graphical Integration Method. The Microsoft Excel package is used to model the Graphical Integration for the three methods.

THE GRADUALLY VARIED FLOW DYNAMIC EQUATION

In deriving the G.V.F dynamic equation, a prismatic channel with a small bed slope is considered. Steady flow, hydrostatic pressure, parallel streamlines and average head loss evaluated over a small reach based on Manning uniform flow are assumed. The G.V.F dynamic equation is given as [3, 5]

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 + \frac{d}{dy}\left(\frac{\alpha v^2}{2g}\right)}$$ (11)

where,
y   is the G.V.F depth,
x   is the distance along the channel,
dy/dx is the water surface slope with respect to bed slope,
S_o is the bed slope,
S_e is the energy slope,
v   is the average velocity,

α is the energy coefficient.

For a circular section eqn. (11) can be written as

$$\frac{d(y\ d_o)}{d(x\ d_o)} = \frac{\left[S_o - \frac{Q^2 * n^2}{\left(m\ d_o^{1/3}\right)^2 \left(A\ d_o^2\right)^2 \left(R\ d_o\right)^{4/3} * d_o^6} \right]}{\left[1 - \frac{\alpha Q^2 \left(T\ d_o\right)}{g \left(A\ d_o^2\right)^3 d_o^5} \right]} \tag{12}$$

Equation (12) represents the exact formulation of the G.V.F dynamic equation and is used to solve the computation of the G.V.F profile length for sustaining (positive) bed slopes.

In order to simplify the computation of the G.V.F profile length, Chow [2] introduced the hydraulic exponents M and N and expressed eqn (12) as

$$\frac{d(y\ d_o)}{d(x\ d_o)} = S_o \frac{\left[1 - \left(\frac{K_n\ d_o^3}{K\ d_o^3}\right)^2 \right]}{\left[1 - \left(\frac{Z_c\ d_o^{2.5}}{Z\ d_o^{2.5}}\right)^2 \right]} \tag{13}$$

Equations (1) and (6) are used to express the critical section factor Z and the conveyance K in terms of hydraulic exponents M and N.
Hence, eqn (13) is modified to

$$\frac{d(y\ d_o)}{d(x\ d_o)} = S_o \frac{\left[1 - \frac{\left(y_n\ d_o\right)^{N_n}}{\left(y\ d_o\right)^N} \right]}{\left[1 - \frac{\left(y_c\ d_o\right)^{M_c}}{\left(y\ d_o\right)^M} \right]} \tag{14}$$

where,
N_n is the exponent evaluated at uniform depth y_n.
N is the exponent evaluated at G.V.F depth y.
M_c is the exponent evaluated at critical depth y_c.
M is the exponent evaluated at G.V.F depth y.

Equation (14) is used to calculate the G.V.F profile length using the hydraulic exponents M and N for constant averaged G.V.F depth based on Chow [2] or the hydraulic exponents

$M(y/d_O)$ and $N(y/d_O)$ for continuously varying G.V.F depth based Zaghloul [15].

COMPUTATION OF THE G.V.F PROFILE LENGTH

Graphical Integration is used to compute the G.V.F profile length based on the various formulation of the G.V.F equations. The G.V.F length based on eqn (12) is compared to the G.V.F length calculated by eqn (14) using the Chow M and N exponents [2] or the Zaghloul modified $M(y/d_O)$ and $N(y/d_O)$ [15].

THE COMPUTER PACKAGE FOR STEADY G.V.F [PACKAGE #1]

A Microsoft Excel spreadsheet is developed to calculate the steady G.V.F profile using the Graphical Integration Method. The model can simulate different flow conditions and various bed slopes. Fixed or minimum relative depth limits can be used in the model. Normal depth based on the Manning uniform flow formula is calculated for constant roughness coefficients. Critical depth based on the minimum specific energy condition is calculated.

The model classifies the flow profile and calculates the steady G.V.F length for positive pipe slopes using constant roughness coefficients. Detailed x-y co-ordinates can be calculated to provide tabular and/or graphical representations of the G.V.F profile. Excel graphics capabilities are used to plot the steady G.V.F profile, the normal and critical depths and the bed slope of the pipe section. The package is provided with a Read me File to help users to feed in data and run the model.

COMPUTATION OF UNSTEADY GRADUALLY VARIED FLOW

Equation of Motion

The momentum equation for flow through a control volume which is fixed in inertial space is [4].

$$\bar{F}_s + \iiint\limits_{cv} \bar{B}\,\rho\,dv = \iint\limits_{cs} \bar{V}\left(\rho\,\bar{V}\bullet d\bar{A}\right) + \frac{\partial}{\partial t}\iiint\limits_{cv}\bar{V}\left(\rho\,dv\right) \qquad (15)$$

where,

$\bar{F}_s$ is the sum of all surface forces acting on control volume,

$\bar{B}$ is the sum of all body forces per unit mass,

ρ is the mass density of water,

dv is the volume element,

$\bar{V}$ is the absolute fluid velocity,

$d\bar{A}$ is the directed area element, positive outward.

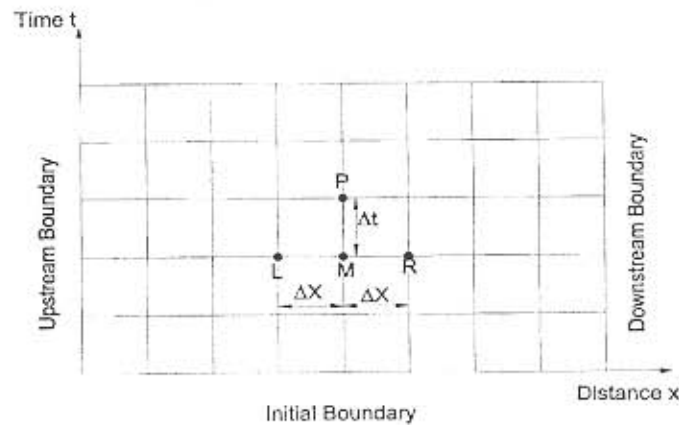the values of V and y at all points on the x-t plane as shown in Fig. 1.

Figure 1   x-t plane - Rectangular Grid

If it is assumed that conditions are known at points L, M and R, unknown values of the dependent variables may be found at P. The substitution of the central and forward differences in the continuity eqn (17), results in

$$Y_P = Y_M + \frac{\Delta t}{2\Delta x} \left[V_M \left(Y_L - Y_R \right) + D_M \left(V_L - V_R \right) \right] \tag{19}$$

and similarly, the momentum eqn (18), appears as

$$\frac{V_P - V_M}{\Delta t} + V_M \frac{V_R - V_L}{2\Delta x} + g \frac{Y_R - Y_L}{2\Delta x} = g \left(S_o - S_e \right) \tag{20}$$

The energy slope S_e is evaluated at point P based on the Manning equation with constant roughness coefficient and a circular pipe section. Thus

$$S_e = \left(\frac{n}{KK} \right)^2 \frac{V_P \left| V_P \right|}{R_P^{4/3}} \tag{21}$$

where,
KK   is constant equals 1.0 for SI unit and 1.49 for British unit,
n   is the Manning roughness coefficient.

Substituting eqn (21) into eqn (20), and solving the resulting quadratic relation will yield:

$$V_p = \frac{-KK^2}{2\,n^2\,g\,\Delta t}\,R_p^{4/3} + \left\{ \frac{KK^2}{n^2\,g\,\Delta t}\,R_p^{4/3} \left[\frac{KK^2}{4\,n^2\,g\,\Delta t}\,R_p^{4/3} \right.\right.$$
$$\left.\left. + V_M \left(1.0 + \frac{\Delta t}{2\Delta x}\left(V_L - V_R\right) + g\,\Delta t\,S_o + g\,\frac{\Delta t}{2\Delta x}\left(Y_L - Y_R\right) \right) \right] \right\}^{1/2} \tag{22}$$

The solution of the finite difference eqns (19) and (22) proceeds by finding a value for each Y_p in the forward time line by eqn (19), substituting this value in eqn (22), and computing each value of V_p in the time line.

INITIAL AND BOUNDARY CONDITIONS

The preceding section set down the equations that deal with the routing procedure in the interior points (Fig. 1). The initial state of the urban drainage system must be known or assumed before a hydraulic routing technique is conducted. The initial condition describes the flow depth, velocity or discharge at all points in the channel at time $t = 0$. Boundary conditions refer to the depth, velocity or discharge at upstream and downstream ends of the channel at all times $t > 0$. Steady uniform flow, backwater conditions, control by weirs or dams, or a non-uniform lateral inflow all are initial and/or boundary effects that must be incorporated into the numerical solution. State of flow condition (subcritical or supercritical) dictate the type of finite difference scheme employed.

1. The upstream Boundary Condition: Equations (19) and (22) are modified to read

$$Y_P = Y_M + \frac{\Delta t}{\Delta x}\left[V_M \left(Y_M - Y_R\right) + D_M \left(V_M - V_R\right) \right] \tag{23}$$

and

$$V_P = \frac{Q}{\text{Area}} = \frac{Q}{f\left(Y_P\right)} \tag{24}$$

Since values of Y_L and V_L are not available to the left of the upstream boundary (see Fig. 1).

The Downstream Boundary Condition: The downstream boundary computation is similarly modified to read

$$Y_P = Y_M + \frac{\Delta t}{\Delta x}\left[V_M \left(Y_L - Y_M\right) + D_M \left(V_L - V_M\right) \right] \tag{25}$$

and

$$V_p = \frac{KK}{n} S_e^{1/2} R^{2/3} \tag{26}$$

which represents Manning equation with constant roughness.

STABILITY OF THE EXPLICIT METHOD

The primary disadvantage of the explicit method is its stability problem. The numerical stability of the solution depends on the relative grid size. A necessary but insufficient condition for the stability of the explicit scheme is the Courant Condition [12], namely

$$\Delta t \leq \frac{\Delta x}{V + CC} \tag{27}$$

where,
CC is the dynamic celerity wave defined as

$$CC = \sqrt{gD} \tag{28}$$

Another condition is known as the friction criteria [14], namely

$$CC \frac{\Delta t}{\Delta x} \leq \sqrt{1 - F \frac{\Delta t}{2}} \tag{29}$$

in which $F = g \, n^2 \, \overline{V} / \left(KK^2 \, \overline{R}^{4/3} \right)$ and the barred quantities represents average flow conditions. This equation indicates that the addition of high frictional losses to a problem that is formulated in this manner may cause instabilities, rather than improve the stability as is quite often assumed. The lesser value of Δt as computed from the Courant condition and the friction criteria, determines the maximum allowable Δt interval.

THE COMPUTER PACKAGE FOR UNSTEADY G.V.F [PACKAGE # 2]

A SECOND Excel package is developed to calculate the spatial and temporal variation of the flood wave in circular pipe with constant roughness. The model utilizes the numerical solution by the Explicit Method. A steady uniform flow condition is considered in the model as the initial condition. The normal discharge Q_o based on the Manning uniform flow formula is calculated for constant roughness coefficient for a given bed slope S_o and pipe diameter d_o.
The total length of the pipe is divided in the model into small reaches Δx. The time step Δt is calculated for the Explicit Method to enforce stability of the numerical solution. The upstream boundary condition is considered as an Inflow Hydrograph and identified in the model in terms

of the peak discharge, the time to peak, the receding time, and the maximum time of simulation. The downstream boundary condition is considered in the model to be the Manning formula. The model calculates the variation of flow discharge, depth and velocity with time at various locations along the pipe. Excel graphics capabilities are used to plot flow hydrographs at the upstream, mid point and downstream sections. A Read me File provide the users with the information needed to feed input data and run the model.

THE STORM WATER MANAGEMENT MODEL (SWMM)

The Environmental Protection Agency Storm Water Management Model is one of the several advanced computer assisted models design to simulate urban storm water runoff. The SWMM is capable of predicting and routing the quantity and quality constituents of urban storm water runoff. The model consists of four functional program blocks, plus a coordinating executive block. The blocks can be overlaid and run sequentially or can be run separately with interfacing data file. The choice of model depends on user needs.

The first of the functional blocks, the Runoff block, simulates the continuous runoff hydrograph and pollutograph for each subcatchment in the drainage basin. Runoff hydrographs are predicted based on an input hyetograph and the physical characteristics of the subcatchment; including area, average slope, degree of imperviousness, overland flow resistance factor, surface storage and overland flow distance. Pollutographs are generated based on the volume of storm runoff and antecedent conditions, including rainfall history, street sweeping data, land use and related data. Runoff flows within each subcatchment may be routed via gutters or pipes, however, all flow from a particular subcatchment must enter one designated manhole for transfer to the second functional block, the transport block.

The transport block represents the combined or separate storm sewer system as a series of conduits joined by manholes. Runoff hydrographs and pollutographs from the various subcatchments enter through the designated manholes and are combined with dry weather flow and rain water infiltration. Real time flow routing of quantity and quality constituents through the transport conduits is accomplished using an iterative, finite difference application of a modified version of the Manning flow equation. Transport elements are allowed to surcharge when they become full and the surcharged flow is stored at the upstream manhole until it can be routed. The transport block contains provisions for modeling lift stations, flow dividers and inline (internal) storage units. The transport block can interface with either the Storage/Treatment block or the Extended Transport block (EXTRAN).

The EXTRAN is a dynamic flow routing model that routes inflow hydrographs through an open channel and/or closed conduit system, computing the time history of flows and head throughout the system. The program solves the full dynamic equation for gradually varied flow using an explicit solution technique to step forward in time. Therefore, the solution time-step is governed by the wave celerity in the shorter channels or conduits in the system. The conceptual representation of the drainage system is based on the "link-node" concept which does not constrain the drainage system to a dendritic form. Links transmit flow from node to node. Properties associated with the links are roughness, length, cross-sectional area, hydraulic radius, and top surface width. The primary dependent variable in the links is the discharge Q Nodes are the storage elements of the system and correspond to manholes or pipe junctions in

the physical system. The variables associated with a node are volume, head, and surface area. The primary dependent variable is head H, which is assumed to be changing in time but constant throughout any one node.

The Storage/Treatment block permits the inclusion of external storage elements and treatment facilities in the modeling scheme. Treatment modes programmed include bar racks, dissolved air flotation systems, fine screens, sedimentation tanks, microstrainers high rate filters, effluent screens and chlorine contact tanks. Treatment facilities are sized based on influent flow rates. Estimated costs can be generated for storage and treatment facilities chosen.

The latest version of PCSWMM is PCSWMM 2000 [6, 7]. This is a 32 bit upgrade to PCSWMM (April 1995). PCSWMM 2000 is a decision support system for the EPA Storm Water Management Model, providing a large array of file management, data file creation output interpretation, and reference tools for the stormwater modeler. Rewritten for the Win '95', '98', Millennium Edition, NT 4.0 and 2000 operating systems, PCSWMM 2000 has taken a new approach to provide a unprecedented level of flexibility and power. Users can develop their own in-house modeling environments using the extensive selection of plug-in tools and PCWMM's seamless integration of any external processes (programs, batch files, macros, etc.).

PCSWMM 2000 is flexible enough to be used with any version of the SWMM engine, and is distributed with the latest version of SWMM 4.4 gu, as well as the official USEPA SWMM 4.3 and later SWMM 4.31 and 4.40 releases. PCSWMM provides full support for all modules of SWMM, including the rain, temperature, runoff, transport, EXTRAN, storage/treatment, combine and statistics modules.

CASE STUDIES

In order to illustrate the simulation of the steady and unsteady G.V.F profiles, three case studies were used. They are namely: (1) the steady G.V.F using the Graphical Integration Method (Excel Package #1), (2) the unsteady G.V.F using the Explicit Method (Excel Package #2), (3) the simulation by the EXTRAN Block of the (PCSWMM2000).

CASE STUDY #1-STEADY G. V. F. [EXCEL PACKAGE#1]

A circular storm drain has a diameter $d_0 = 1.50$ m and carries a discharge $Q = 0.5$ m^3/s. The Mannings roughness coefficient n = 0.015. The coefficient of energy $\alpha = 1.10$. Use the Graphical Integration Method to compute the steady G.V.F length generated for various bed slopes and G.V.F depth limits (minimum or fixed) as given in Tables 1 and 2. Use eqn. (12) for exact calculation of the G.V.F profile length and compared the results with the calculation based on eqn. (14) for constant hydraulic exponents (M, N) and varied hydraulic exponents ($M(y/d_0)$, $N(y/d_0)$). Use the steady Excel Package #1 for the calculation and plotting the results.

Results of Simulation

Input data of Table 1 for various bed slopes and minimum G.V.F depth limits are used to calculate the steady G.V.F length using the Graphical Integration Method. The results are

summarized in the same Table 1. Input data of Table 2 for various bed slopes and fixed G.V.F depth limits are used to calculate the steady G.V.F length using the Graphical Integration Method and the results are summarized in the same Table 2. Figures 2 to 4 show plots of the M_1, C_3 and S_2 steady G.V.F profiles. The critical depth, the normal depth and the bed slope are marked on each plot.
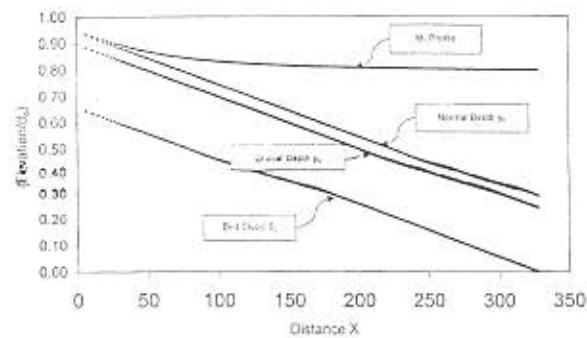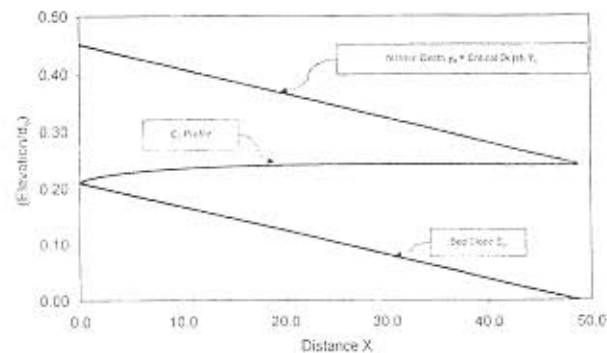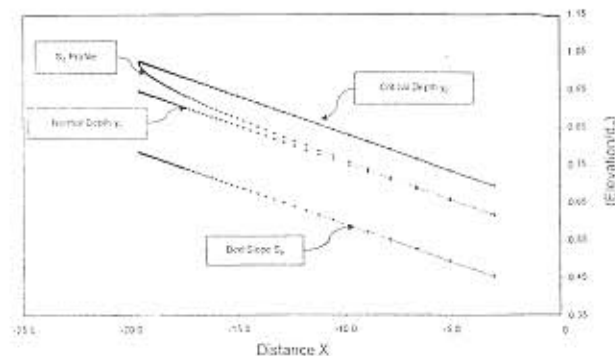

Figure 2.

Figure 3.

Figure 4.

CASE STUDY #2-UNSTEADY G. V. F. [EXCEL PACKAGE#2]

A circular storm drain has a diameter $d_0 = 1.50$ m and length L = 1 kilometer. The Manning

roughness $n = 0.015$ and the bed slope $So = 0.002$. The initial condition is taken as a steady uniform flow depth $= 0.30$ m. The upstream boundary condition is taken as an Inflow hydrograph, Fig. 5. The uniform discharge calculated based on Manning formula for the above steady flow depth $Q_0 = 0.24$ m^3/s is kept constant for 5.0 minutes. The discharge is increased linearly to $Q_{max} = 2.70$ m^3/s over a period 20 minutes, then decreases linearly to the initial discharge Q_0 in an additional period of 40 minutes, and is kept constant for another 5.0 minutes. The total simulation time $T_{max} = 70$ minutes (Fig. 5). The downstream boundary condition is taken as Manning uniform flow formula. Calculate the spatial and temporal variations of the flood wave using the Explicit Method use the unsteady G.V.F Excel Package #2 for the simulation of the wave propagation through the drain and plotting the routed hydrographs at various distances.
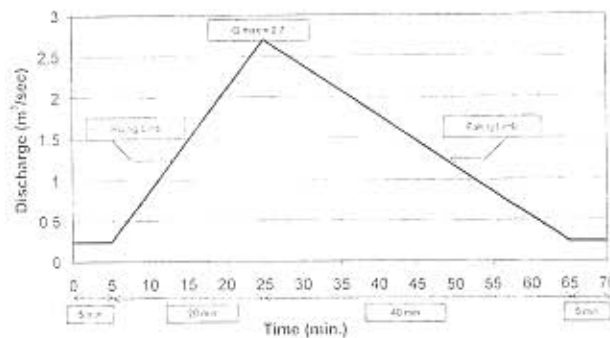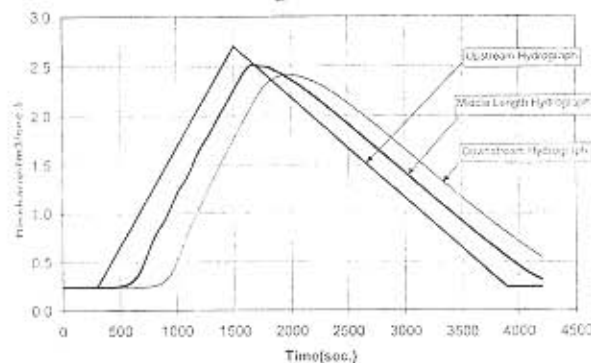

Figure 5.

Figure 6.

Results of Simulation

The Explicit Method is adopted to solve for the routing of the flood wave through the circular channel. The routed hydrographs are plotted at the upstream, mid point and downstream section of the pipe. Figure 6 shows the three routed hydrographs.

CASE STUDY #3-PCSWMM2000- EXTRAN SIMULATION

Use the PCSWMM2000-EXTRAN Block for the simulation of the wave propagation given

in Case study #2. Compare the routed hydrographs resulted from the EXTRAN Block and the unsteady G.V.F Excel Package #2.

Results of Simulation

The EXTRAN block is used to simulate the flow routing through the given drain. Figure 7 shows the routed hydrographs at the upstream, mid point and downstream section of the pipe for constant roughness conditions.
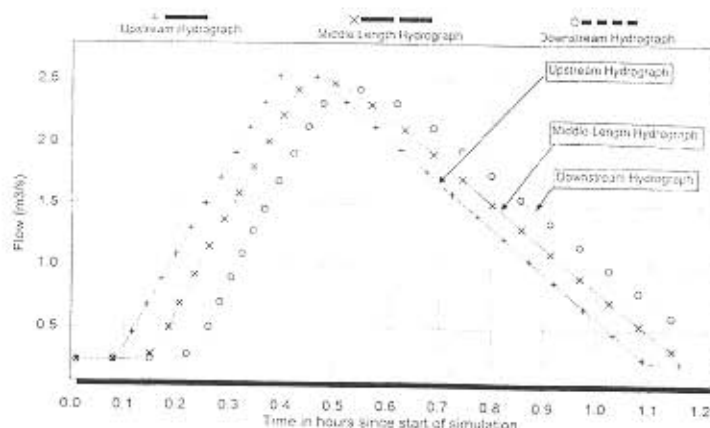

Figure 7.

CONCLUDING REMARKS

Steady G.V.F

The steady G.V.F length calculated using the Graphical Integration Method. The dynamic G.V.F equation is used to calculate the exact length of the G.V.F profile. The G.V.F length is also calculated based on constant and variable hydraulic Exponents and is compared to the exact length from the dynamic G.V.F equation for various bed slopes and G.V.F depth limits.

The results of the calculated G.V.F profile length using the variable hydraulic exponents equation are closer to the G.V.F length calculated based on the exact formulation of the G.V.F dynamic equation. The percentage difference ranges from 0.67% to 8.72% for various bed slopes and G.V.F depth limits. The calculated G.V.F profile length using constant hydraulic exponents resulted in a wider values with percentage difference ranges from 0.16% to 25.59% (see Tables 1 and 2). A computer package was developed for the steady G.V.F length calculation using the Microsoft Excel spreadsheet (Steady G.V.F Package #1).

Unsteady G.V.F

The Explicit Method simulation provides comparable results to the (PCSWMM2000-EXTRAN Block). The routed hydrographs at the Mid point and the Downstream sections are similar for both models in terms of peak discharge and lag time. The Explicit Method simulation provides stable results using the Microsoft Excel Package #2.

Both Excel packages 1 and 2 are user friendly. Two Read me Files are supplied in order to help the users to feed input data and run the programs. The Excel packages are posted on the

Internet Website Location http://briefcase.yahoo.com/civil_engineering2001.

The developed computer model for steady and unsteady G.V.F simulations will equip design engineers involved in storm drainage and wastewater flow systems with a powerful and efficient design tool.

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Table 1. G. V. F length for graphical intergration method using exact equation and M & N (constant or variable) for minimum depth limits

Case	Bed Slop Classification	Bed Slop	Normal Depth (y_n/d_o)	Critical Depth (y_n/d_o)	Profile	Y G.V.F Limits		Profile Length			% Length Variation*	
						Y_1/d_o	Y_2/d_o	Constant M, N	Variable M, N	Exact Equation	L (M, N) Constant/L (exact)	L (M, N) Variable/L (exact)
I	Mild	0.002	0.289	0.242	M1	1.01Yc/do	0.8	312.98	313.15	328.23	4.64	4.60
					M2	0.99Yc/do	1.01Yc/do	25.64	26.74	27.30	6.10	2.06
					M3	0.001	0.99Yc/do	30.98	37.58	41.11	24.64	8.57
II	Critical	0.00403	0.289	0.242	C1	1.01Yc/do	0.8	135.23	136.21	138.12	2.09	1.38
					C3	0.001	0.99Yc/do	39.96	46.64	48.71	17.98	4.26
III	Steep	0.02	0.163	0.242	S1	1.01Yc/do	0.8	24.28	24.49	24.32	0.16	0.67
					S2	0.99Yc/do	1.01Yc/do	18.90	19.78	19.61	3.60	0.87
					S3	0.001	0.99Yc/do	42.24	46.98	46.22	8.61	1.65

$$*\%\text{Length Variation} = \left| \frac{\{L(exact) - L(M,N) cons\tan t\ or\ variable\}}{L(exact)} \right| *100$$

Table 2. G. V. F length for graphical intergration method using exact equation and M & N (constant or variable) for fixed depth limits

Case	Bed Slop Classifica tion	Bed Slop	Normal Depth (y_n/d_o)	Critical Depth (y_c/d_o)	Profile	Y_1/d_o	Y_2/d_o	Constant M, N	Variable M, N	Exact Equation	L (M, N) Constant/L (exact)	L (M, N) Variable/L (exact)
							Y G.V.F Limits	Profile Length			% Length Variation*	
I	Mild	0.002	0.289	0.242	M1	0.4	0.8	208.13	205.84	213.08	2.32	3.40
					M2	0.35	0.3	48.13	49.36	52.56	8.42	6.08
					M3	0.01	0.2	26.69	32.74	35.87	25.59	8.72
II	Critical	0.00403	0.289	0.242	C1	0.4	0.8	98.51	98.51	99.74	1.22	1.23
					C3	0.01	0.2	30.69	36.57	38.26	19.78	4.44
III	Steep	0.02	0.163	0.242	S1	0.3	0.8	23.20	23.38	23.24	0.16	0.62
					S2	0.23	0.18	4.96	5.16	5.10	2.60	1.18
					S3	0.01	0.1	13.53	16.10	15.91	14.92	1.22

$$*\%\text{Length Variation} = \left| \frac{\{L(exact) - L(M,N)(cons\tan t\ or\ variable\}}{L(exact)} *100 \right|$$